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Improving the visual perception of sonar signals with stochastic resonance

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NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

IMPROVING THE VISUAL PERCEPTION OF SONAR SIGNALS WITH STOCHASTIC RESONANCE

by

Allison P. Went

June 2007

Thesis Advisor: Daphne Kapolka

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IMPROVING THE VISUAL PERCEPTION OF SONAR SIGNALS WITH STOCHASTIC RESONANCE

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Submitted in partial fulfillment of the
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I. INTRODUCTION

A. MOTIVATION

The purpose of this thesis is to examine the possibility of using stochastic resonance (SR) to aid in the detection of weak tonals in passive sonar displays. This topic was suggested by G. Scott Peacock of The Applied Physics Laboratory – John Hopkins University (APL-JHU) as the first project of a collaboration between APL-JHU and the Naval Postgraduate School (NPS). APL-JHU has been an integral member for many years in the Advanced Processing Build (APB) program. The APB program is run by the Program Executive Officer - Integrated Warfare Systems. This program seeks to capitalize on the speed and memory improvements available in commercial computer hardware to shorten the timeline required to field new sonar processing algorithms in the fleet. The APB program is supported by a large number of government and university laboratories, warfare centers, and private industry. Developers use "open" data sets collected by fleet sonar systems to develop new algorithms. The ideas and results are subject to a peer review process, and algorithms are tested on previously unavailable or "closed" data sources prior to acceptance for the next sonar processor build. This process has been responsible for significant improvements in fleet sonar performance. NPS has not been a participant in the APB process; however, it is hoped that this collaboration with APL-JHU will serve as a starting point for NPS students and faculty to both learn from and contribute to this important program. From the broad topic of stochastic resonance, the issue of image improvement was chosen for this thesis.

B. LOFARGRAMS

LOFAR stands for "Low Frequency Analysis and Recording." On U.S. Navy ships, 3-bit LOFARgrams are used to display data obtained from passive

sonar. On a screen, a LOFARgram has a vertical axis representing time and a horizontal axis representing frequency. If the signal spectrum level is high at a certain frequency, the pixels corresponding to that frequency in the LOFARgram will be dark, while if the spectrum level is low, the pixels will be light. As time passes, old information is dropped from the bottom of the display, and new information is added to the top. This is often referred to as a "waterfall display." In this thesis, a grayscale will be used, where the lowest value represents white and the highest value represents black. LOFARgrams are requantized on a non-linear scale in order to accentuate the differences between low level signals. Due to the requantization, two low-level signals that are relatively similar in signal strength may fall into two different bins, while two high-level signals must be far apart in signal strength or they will fall into the same bin. Figure 1 shows a sample LOFARgram. The signal was collected from a passing airplane using a simple recording device. The LOFARgram was created using the General Purpose Acoustic Analysis Tool (GPAAT) from APL-JHU. This software was written in MATLAB by G. Scott Peacock and his group at APL-JHU to perform the basic signal processing techniques used in fielded Navy sonar processors for both broadband and narrowband signals. It also includes many of the tools used by acoustic analysts for signal analysis.

Figure 1. Sample grayscale LOFARgram

In a tactical situation, it is important for operators to detect targets of interest as soon as possible. Observing tonals in the LOFARgram may enable

an operator to detect, classify, and sometimes even identify a contact. Background noise in collected signals makes it difficult for operators to observe weak signals. The goal of this research project is to improve the detectability of low-level tonals in LOFARgrams by reducing the negative effects of background noise.

C. STOCHASTIC RESONANCE

Stochastic resonance was first proposed in the early 1980's as an explanation for the enhancement of weak periodic signals by noise. The word "stochastic" refers to anything involving a random variable. In this case, the random variable is the noise added. The word "resonance" is used because the goal is to apply exactly the right amount of noise in order to create an optimal, or "resonant," effect.

There are three basic ingredients needed to create the phenomenon: a threshold, a weak periodic signal, and a source of noise that is either inherent in the system or purposefully added. Where a weak periodic signal alone might be insufficient to exceed the threshold, the addition of the right amount of noise to the signal may enable the signal to exceed the threshold and the effect to be periodically manifested.

There have been many different techniques proposed for applying the phenomenon of stochastic resonance to improving the signal-to-noise ratio of weak sinusoids. These techniques are all counter-intuitive in that they seek to improve the detectability of signals by adding noise. An overview of these techniques will be provided in Chapter II. In this thesis we investigate the use of stochastic resonance (SR) as an image processing technique to improve the visual perception of sonar signals in LOFARgrams. A brief explanation of how stochastic resonance can be used as an image processing technique is provided in this introduction and a more detailed analysis is given when we discuss the theory in Chapter III.

The starting point for the SR image processing procedure is a noisy signal that has been processed into the time-frequency plot of a LOFARgram. For a simple example, consider the image in Figure 2. This image increases in pixel value from 0 (white) on the left to 256 (black) on the right. In addition, the image includes a horizontal gray line of constant value 128.

Figure 2. Grayscale example image with no background noise.

Figure 3 shows the same image with background noise added.

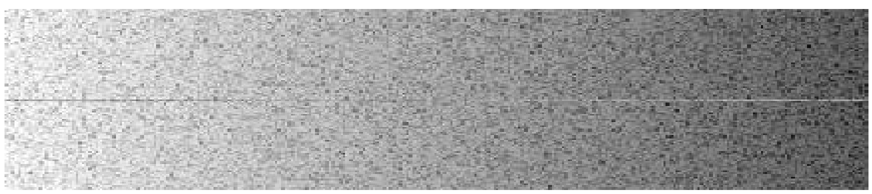

Figure 3. Example image with background noise

After adding noise to each pixel in an image, a threshold is applied to the image. If the value of the pixel crosses the threshold, it is replaced with the maximum pixel value, otherwise it is replaced with the minimum value. The result is a purely black and white image. This procedure is repeated multiple times, and the black and white images are either averaged together to get the final result, or shown in sequence as a movie. In this thesis, we experimented mainly with averaged SR images, although we also made some movies to observe. Figure 4 shows our sample image after it has gone through the SR process. If we can show that the resulting image is better than the noisy image, the process has been successful. In Chapter IV, we will discuss the methods used for measuring which image is "better."

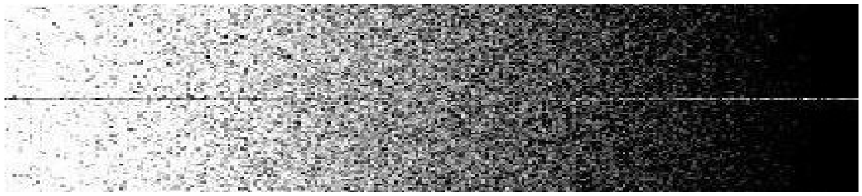

Figure 4. Example image after adding noise, applying threshold, and averaging 20 images

The key element of this method is choosing the correct noise and threshold values. These values must be chosen so that low level tonals will be more likely to cross the threshold than background noise. Adding too much noise may make the original image unrecognizable, while adding too little noise may not extract weak signals from background noise. Choosing a threshold that is too low will result in a dark image, where background noise and signals both cross the threshold. Too low of a threshold will produce a very light image, with tonals too dim to see.

D. CONTRIBUTION OF THIS THESIS

This thesis investigates the possibility of using stochastic resonance to improve the detection of low-level signals in LOFARgrams. Although previous researchers have suggested applying the SR process to sonar signals, this research project is the first to apply SR image processing to realistic 3-bit LOFARgrams. It is also provides a more complete theoretical analysis of the process than previously reported. This analysis explains why the addition of noise cannot improve the signal-to-noise ratio of the averaged images. However, it is shown that under certain conditions, the signal-to-noise ratio (SNR) of the original image can be preserved. The results of this thesis suggest that any improvement in low-level tonal detectability can only be accomplished by presenting changing images to the operator.

E. THESIS OUTLINE

The outline of this thesis is as follows. Chapter II investigates previous studies involving stochastic resonance. Chapter III discusses the theory behind the process. Chapter IV presents the simulations conducted for this study along with the images before and after applying the stochastic resonance process. Chapter V summarizes the results and offers suggestions for future research and applications.

II. BACKGROUND

A 1998 review of stochastic resonance published by Gammaitoni, Hänggi, Jung, and Marchesoni in the Reviews of Modern Physics references literally hundreds of papers investigating the phenomenon of stochastic resonance since it was first proposed in the early 1980's. This trend is still going strong. Stochastic resonance has been invoked to explain such diverse phenomena as the periodicity of ice ages, the behavior of bistable ring lasers, and neuron firing rates. For the purpose of this thesis, however, the focus will be on stochastic resonance as a technique in image processing.

This chapter will summarize the results of experiments and simulations done by previous researchers and comment on the possibility of applying these results to the specific case of LOFARgrams.

A. VISUAL PERCEPTION OF STOCHASTIC RESONANCE

The primary article that led to this study is entitled "Visual Perception of Stochastic Resonance" by Enrico Simonotto et al. It was published the in the Physical Review Letters in February 1997. The experiment described in this article demonstrates how SR can be used to increase a person's ability to perceive very low-level stripes in an image. The researchers began by creating a noise-free grayscale strip with the pixel values determined by Asin(1/x)+128. A plot of this equation and the resulting strip are presented in Figure 5.

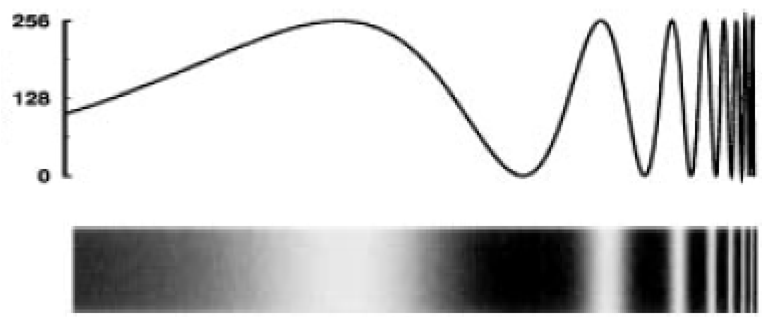

Figure 5. Plot of Asin(1/x)+128 and noise-free grayscale strip (From Simonotto 1997)

Next, the amplitude A of the function was altered in order to generate strips with decreasing contrast. These strips were then subjected to the SR process. Ten different values for the standard deviation σ of Gaussian distributed noise were used to add noise to the original image. The resulting noisy image was then compared to a constant threshold of 150. Thus, ten sets of strips were generated. Each set had a constant value for σ and Δ, and contained strips with seven different values of A, decreasing from 128 to 0. Throughout the experiment, the strips with different realizations of the added noise were shown on a high speed computer monitor at a frame rate of 60 Hz.

Each subject was presented one set of strips at a time, with contrast decreasing from bottom to top. A sample set of 3 strips is shown in Figure 6.

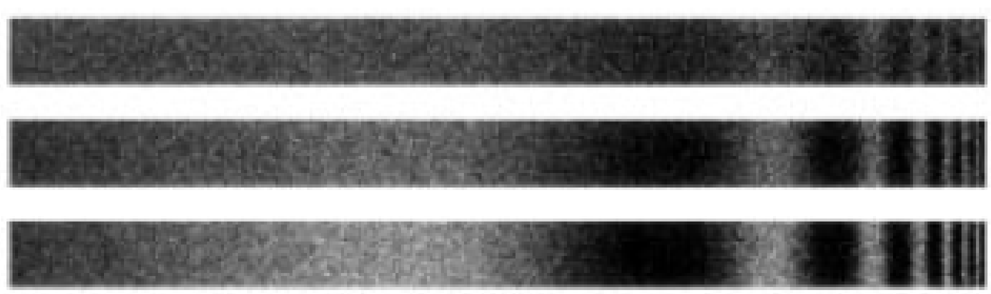

Figure 6. Three sample strips with decreasing contrast A=28, 78, and 128 from top to bottom, processed with σ=250 and Δ=150 (From Simonotto 1997)

The subject was told to count up from the bottom strip until he or she could no longer distinguished a specified feature of the image, such as one of the thin bands near the end of the strip. The smallest amplitude for which they were able to distinguish the given feature was their “perceptive contrast threshold,” A_{th}, for that amount of added noise. This procedure was repeated 100 times for each subject, so that the subject viewed each of the ten sets of strips ten times. Figure 7 displays the result of this experiment for one subject. It is a plot of A_{th} vs. σ, with the error bars representing the standard deviations of the 10 A_{th} values found for each value of σ. The dashed curve is the plot of an equation determined from threshold SR theory, which will not be discussed in this thesis.

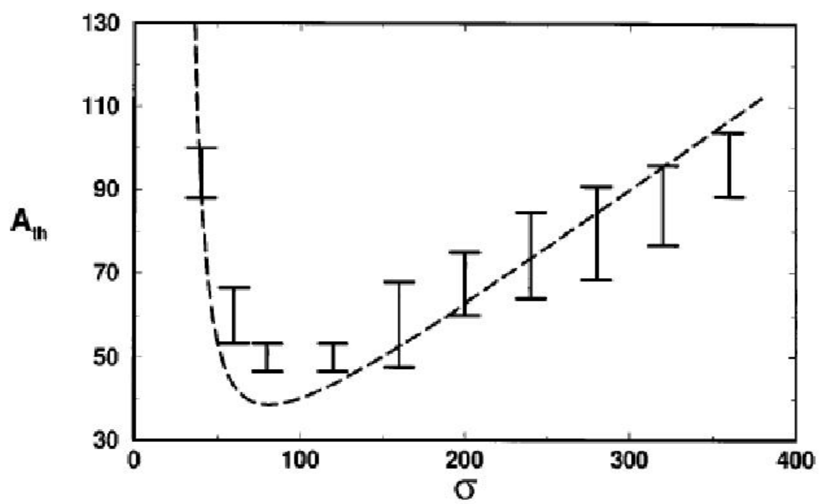

Figure 7. Perceptive contrast threshold A_{th} vs. noise intensity σ for one subject (From Simonotto 1997)

A notable result from this experiment is that, up to a point, adding noise enabled the subject to distinguish features in the image when the contrast was diminished. The optimal σ value for this case appears to be around 70.

Before attempting to apply the results of this experiment to LOFARgrams, there is one major difference that must be considered. The starting point for this experiment was a noise-free image. When dealing with sonar signals, there is

always some amount of background noise. Simonotto's experiment demonstrates that an SR image with more noise may be better than one with less noise, however it does not mention whether or not an original noisy image can be improved upon using this method.

B. IMAGE VISUALIZATION AND EXPLANATION

Another article used for background research in this study is entitled "Stochastic Resonance of a Threshold Detector: Image Visualization and Explanation," by Robert Marks et al. It was published in the IEEE International Symposium on Circuit and Systems Proceedings in May, 2002. This aim of this article is to optimize the appearance of an image using stochastic resonance. The image used in this study was a photograph of a man's face. The researchers used uniform noise for their experiments, and developed an algorithm for calculating the optimal noise and threshold values.

In this experiment, the researchers used a scale from 0 (white) to 1 (black). They chose the threshold to be $\Delta = \frac{1}{2}$. With these conditions, the expectation value of each pixel after the SR process was calculated for different amounts of noise. When uniform noise was added from $-\frac{1}{2}$ to $\frac{1}{2}$, the expectation value of each pixel was equal to its original value. Therefore, when many different SR images were averaged together, the final product converged to the original image. This is illustrated in Figure 8.

(a)

(b)

Figure 8. (a) The original image used for the SR experiment. (b) Multiple realizations of SR images averaged together. From left to right, the images represent 2, 5, 10, and 50 averaged images. With more realizations, the final image converges to the original. (From Marks 2002)

The convergence demonstration illustrates why it is important to average multiple images. This article also demonstrates that with the right threshold and noise level, it is possible to approach the original image using SR. The remaining question is if the original image can be improved upon using the same technique.

One contribution from this article is the suggestion of using uniform random noise instead of Gaussian. This sets boundaries to how much the value of one pixel can vary. In the article, each pixel could only vary as much as ½. Since the threshold was set to ½, a pixel with an initial value of 0 could never surpass the threshold. When using Gaussian noise, there are no strict boundaries to the variation in each pixel. This makes the expectation value of a pixel after applying the threshold and the appearance of the final image more

difficult to calculate. This article seems to imply that the improvement obtained in Simonotto's work resulted from the averaging process, but correspondence with one of the authors [Fox 2007] revealed that they remained open to the possibility that an advantage may be realized through the moving images. This area is still under active investigation. [Mitaim and Kosko 2004]

C. IMAGE DENOISING USING STOCHASTIC RESONANCE

A third article used for background for this study is entitled "Image Denoising Using Stochastic Resonance." It was written by Rajib Kumar Jha, P.K. Biswas, and B.N. Chatterji of the Indian Institute of Technology, and published in the Proceedings of the International Conference on Cognition and Recognition in 2003. The main contribution of this article was the suggestion of measuring the quality of an image by calculating its peak signal-to-noise-ratio, or PSNR. The definition of PSNR is given in Eqn. **1**.

$$PSNR = 10\log_{10}\left[\frac{P^2}{MSE}\right] \qquad \textbf{Eqn. 1}$$

where P is the maximum value of the grayscale and MSE is defined by Eqn. **2**.

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\left\| I(i,j) - K(i,j) \right\|^2 \qquad \textbf{Eqn. 2}$$

In Eqn. **2**, m and n are the dimensions of the image, *I* is the original noise-free image, and *K* is the final SR image. Thus, the closer the SR image resembles the noise-free original image, the higher the PSNR will be. The image with the greatest PSNR is the optimal SR result, and from this, the optimal noise and threshold values can be found. The researchers applied the SR process and the measurement of PSNR to manipulate the image in Figure 9. The image on the right is the closest possible SR approximation to the image on the left, given the noisy image in the center.

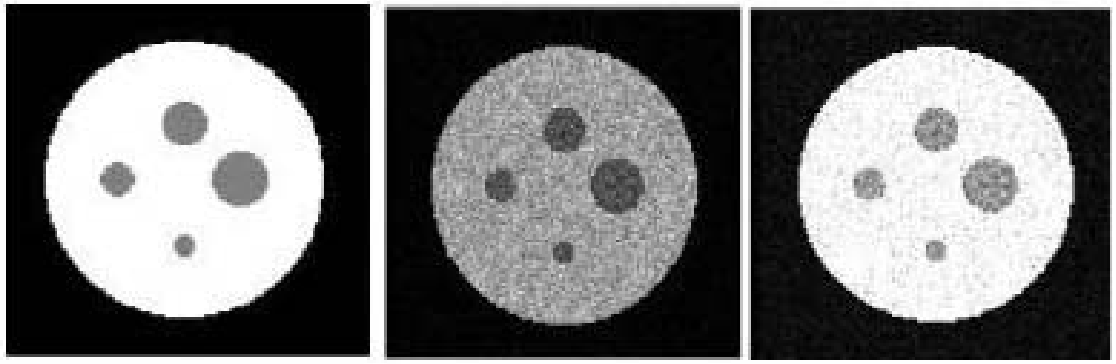

Figure 9. Optimal SR results found from a calculation of PSNR. From left to right are the original noise-free image, the noisy image, and the final image. (From Jha 2003)

PSNR is a valid way of measuring the quality of an approximation to an image. However, when dealing with LOFARgrams, the original noise-free image is not available to compare with the final product. Therefore, it is necessary to find another way to calculate the optimal noise and threshold values for the SR process. This will be discussed further in Chapter IV.

D. STUDIES DONE BY THE CHINESE ACADEMY OF THE SCIENCES

Several articles have been published by the Institute of Acoustics at the Chinese Academy of the Sciences on the subject of stochastic resonance. Two of these papers mentioned the possibility of applying SR to LOFAR displays in order to extract weak lines from the images. In the article entitled “A SR-Based Radon Transform To Extract Weak Lines from Noise Images,” an example is given of lines in a noisy image that are enhanced by a SR-based Radon transform. However, specifics are not given about the strength of the noisy signal, the amount of noise added, and the threshold applied. Also, the improvement of the image was not measured quantitatively (Ye 2003). In the second article, entitled “Image Enhancement Using Stochastic Resonance,” another visual example is provided. The enhanced SR image shows much improvement over the noisy image, but again, the details regarding the scale used and the exact process followed are not included (Ye 2004).

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III. THEORY

This chapter explains the mathematics behind the SR process and calculates the expected values of pixels that are subjected to the SR algorithm. It demonstrates how applying a threshold can increase the SNR of in image, whereas adding noise does not result in any additional improvement.

A. PROBABILITY DISTRIBUTIONS

Part of the stochastic resonance process involves adding noise to every pixel in an image. In this study, two different noise distributions were experimented with: the Gaussian distribution and the Uniform distribution.

1. The Gaussian Distribution

The Gaussian Distribution, also known as the normal distribution, is a symmetric distribution with a probability density function, or pdf, resembling a "bell" curve (Devore 2004). For all distributions, the area enclosed by the pdf must be equal to 1. The standard normal distribution has a mean, μ, equal to zero and a standard deviation, σ, of 1. In this thesis, Gaussian noise added to images will always have a mean of zero. The variable that controls how much noise is added is the standard deviation. A high standard deviation results in a more spread out distribution. Therefore as σ increases, the difference between the initial and final values of each pixel will increase on average. Figure 10 shows the pdfs for Gaussian distributions with three different values of σ.

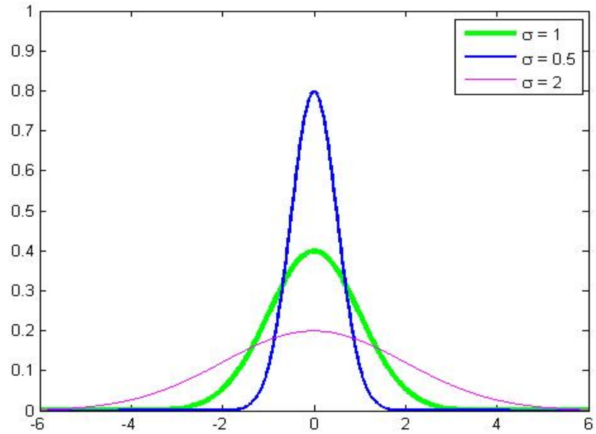


Figure 10. Probability Density Functions of Normal Distributions with Varying Standard Deviation

Another way to describe a distribution is by its cumulative distribution function, or cdf. The cdf $F_\xi(n) = \Pr(\xi \le n)$ represents the probability that the value of the random variable ξ will be less than or equal to n. Figure 11 shows the cdfs for the three pdfs displayed above.

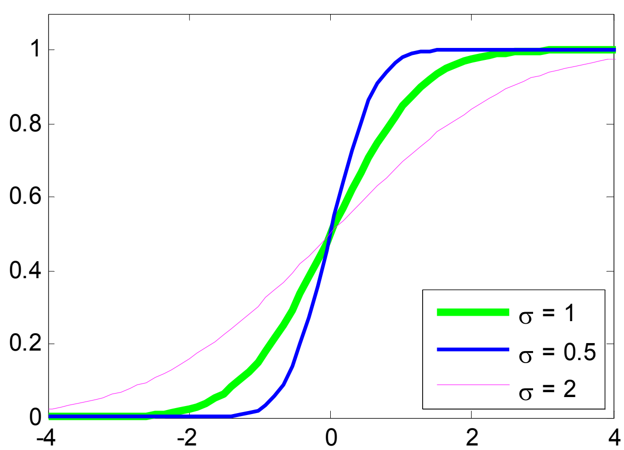


Figure 11. Cumulative Distribution Functions of Normal Distributions with Varying Standard Deviation

2. The Uniform Distribution

In a uniform distribution, every value within the boundaries is equally probable, and every other value has a probability of 0 (Devore 2004). In this thesis, all uniform distributions will have a mean of zero. The parameter used to define the limits of the distribution will be α. For example, a uniform distribution with α =1 will have equal probability for all values between -1 and 1. As in the Gaussian distribution, increasing the value of α increases the average difference between the final and initial values of a pixel. Figure 12 shows the pdfs for uniform distributions with three different values of α.

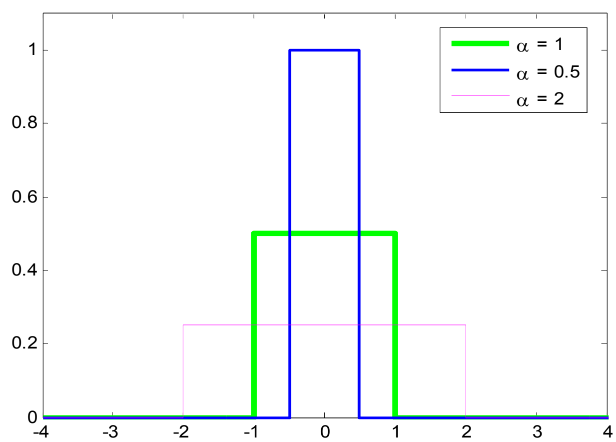


Figure 12. Probability Density Functions of Uniform Distributions with Varying Parameters

Figure 13 shows the cdfs for the three pdfs displayed above.

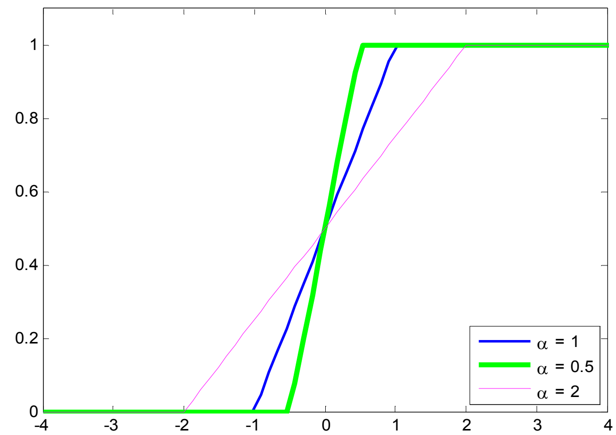


Figure 13. Cumulative Distribution Functions of Uniform Distributions with Varying Parameters

B. STOCHASTIC RESONANCE APPLIED TO IMAGES

Let x be a random variable which corresponds to the original level of a pixel in an image. Assume that the probability density function of x in the case where the pixel contains nothing but noise is given by $f_o(x)$. The mean of this distribution is μ_o. If there is signal present in the pixel with a level x_s, $f_o(x)$ will be displaced to the right by an amount equal to the signal level. The new pdf is $f_1(x)$, where $f_1(x) = f_o(x - x_s)$. The mean of the distribution with signal present is μ_1. These two pdf's are shown below for the case where the random variable has a Gaussian distribution.

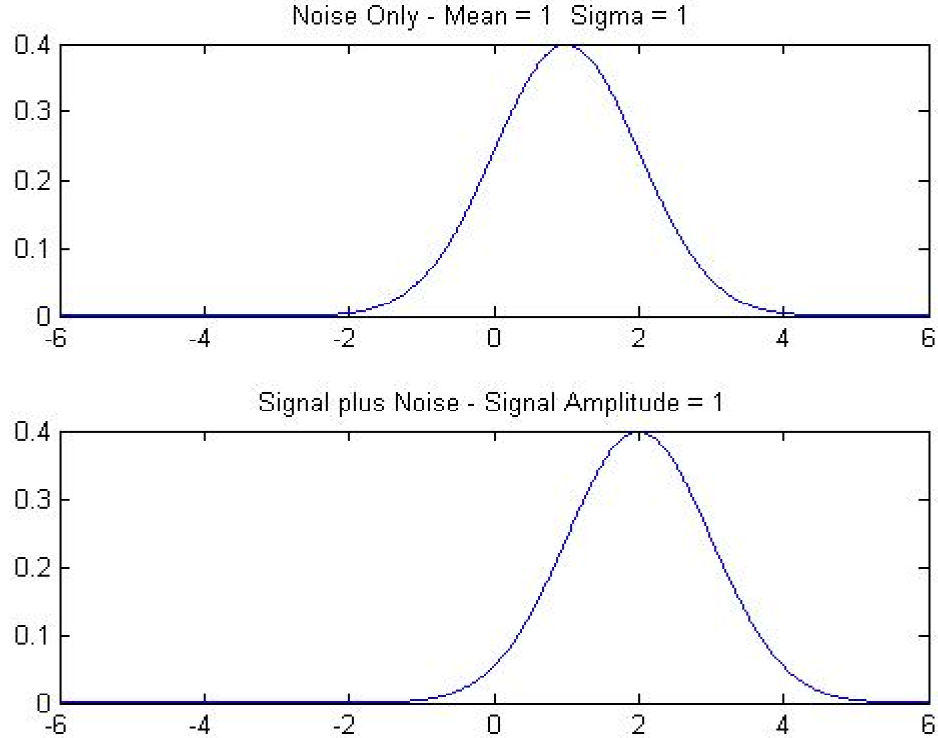


Figure 14. PDFs of a) Noise only and b) Signal plus noise

The stochastic resonance algorithm involves adding additional noise to each pixel and then applying a threshold to the resulting pixel levels of the image. If the pixel exceeds Δ, it is changed to the maximum pixel value, and if it is less than Δ, the pixel value is changed to zero. Before investigating the effect of adding additional noise prior to thresholding, it is important to understand the effect of thresholding in the absence of additional noise. Letting the value of the pixel resulting from the thresholding procedure be z, the expected value of z is given by the probability that the pixel level exceeds the threshold. This can be found directly from the cumulative distribution functionof the pixel level. The expected value of the pixel, $E[z]$ is given by:

$$E[z] = \Pr[x \geq \Delta] = 1 - \Pr[x \leq \Delta] = 1 - F(\Delta) \qquad \textbf{Eqn. 3}$$

where $F(\Delta)$ would be replaced by $F_o(\Delta)$ in the absence of signal or $F_1(\Delta)$ if the signal were present.

To examine how the expected pixel value will vary after thresholding when signal is present versus when it is not present, let z_o equal the expected value when signal is not present and z_1 equal the expected value when signal is present. These expected values can be expressed in terms of the corresponding cdf's as:

$$z_o = 1 - F_o(\Delta) \qquad \textbf{Eqn. 4}$$

$$z_1 = 1 - F_1(\Delta) = 1 - F_o(\Delta - x_s) \qquad \textbf{Eqn. 5}$$

The figure below shows a graph of the expected pixel value as a function of threshold for the situation where only noise is present and where both signal and noise are present. Notice that the SNR can show improvement just on the basis of thresholding. As an example, when a threshold of 2 is applied in this case, the SNR goes from $\left(\mu_1 / \mu_o \right)^2 = 4$ to approximately $\left(z_1 / z_o \right)^2 = \left(0.5 / 0.15 \right)^2 \cong 11$.

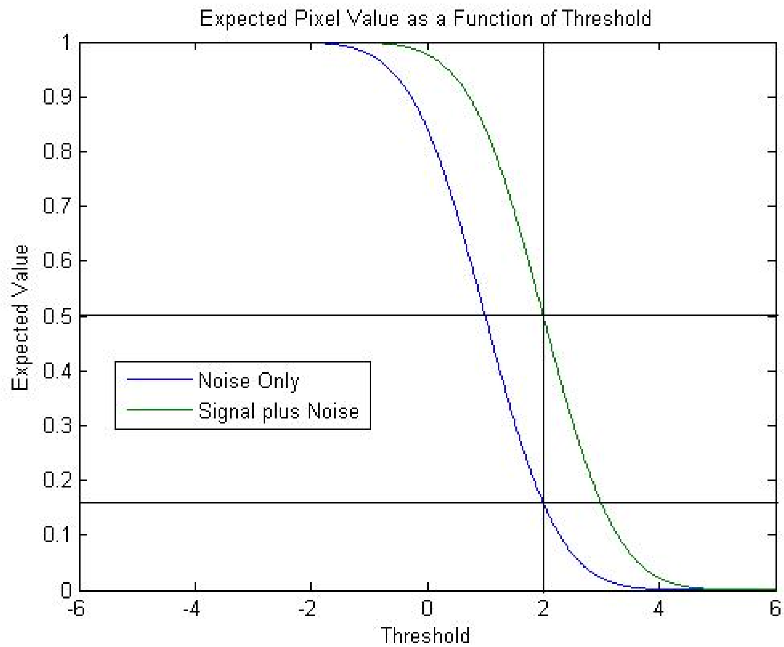


Figure 15. Expected Pixel Value as a Function of Threshold

If the threshold is set at the mean of the signal level, the pixel will be changed to one 50% of the time. These results are identical to those of a threshold detector where the expected value of the pixel in the presence of signal corresponds to the probability of detection (P_d) and the expected value of the pixel in the absence of signal corresponds to the probability of false alarm (P_{fa}). Thus one expects the same type of behavior for the pixel level as one observes on a Receiver Operator Characteristic (ROC) curve, namely that both P_d and P_{fa} decrease with increasing threshold levels and that P_d can only be raised relative to a given P_{fa} by increasing the signal level. Notice also that the resulting pixel level can easily be scaled from the zero to one to any other arbitrary scale simply by multiplying the result by the maximum level desired.

Now let's consider what happens if the stochastic resonance algorithm is used. First noise is added to each pixel in the image. Let's call the random variable corresponding to the noise added, ξ. The probability density function of

the added noise is $g(\xi)$. It is assumed that the added noise is independent from the original pixel value and has a mean of zero. The new random variable corresponding to the original pixel level plus the added noise is $y = x + \xi$. This random variable will have a new pdf, $h(y)$, and a new cdf, $H(y)$. Since the mean of $g(\xi)$ is zero, the mean of the random variable y is the same as the mean of the original random variable, x. However, the second moment of $h(y)$ about its mean will always be greater than the original. To see this consider the expectation value of $(y-\mu_y)^2$.

$$E\left[\left(y-\mu_y\right)^2\right] = E\left[y^2-\mu_y^2\right] = E\left[\left(x+\xi\right)^2-\mu_x^2\right] = E\left[x^2+2x\xi+\xi^2-\mu_x^2\right] \quad \textbf{Eqn. 6}$$

This simplifies to:

$$E\left[\left(y-\mu_y\right)^2\right] = E\left[x^2+2x\xi+\xi^2-\mu_x^2\right] = E\left[\left(x-\mu_x\right)^2\right] + E\left[\xi^2\right] \quad \textbf{Eqn. 7}$$

Thus the second moment about the mean, i.e. the variance of the new random variable, y, is always greater than the original by an amount equal to the variance of the added noise. The graph below shows how the expected value of the pixel changes with the threshold when additional Gaussian noise is added to the original image to increase the standard deviation to two.

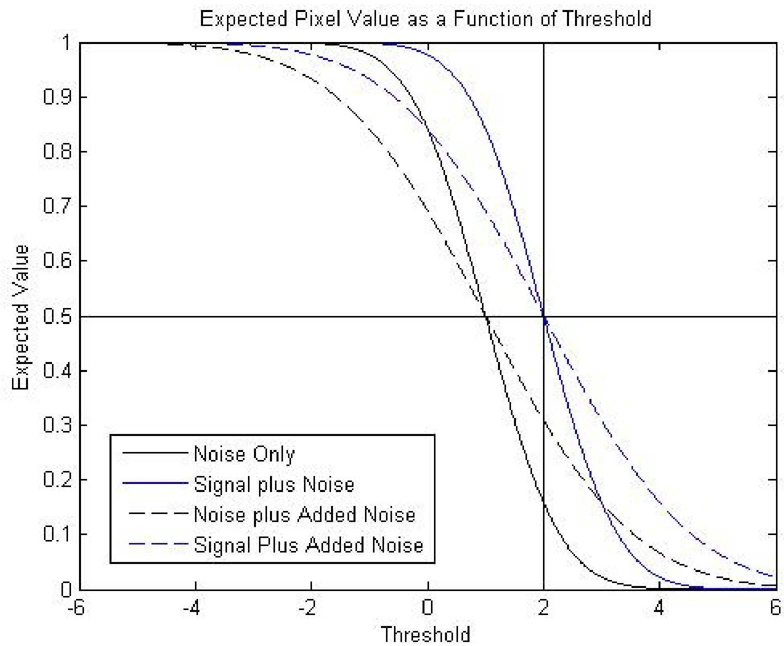


Figure 16. Expected Pixel Value as a Function of Threshold

It is clear from both the diagram and from the analogy to the ROC curve that the addition of noise followed by thresholding results in a lower signal to noise ratio than when no additional noise is added. Although this argument used Gaussian noise, the addition of any noise distribution would have the same effect. The lack of improvement in SNR does not necessarily mean that low level signals can not be made more detectable by a human operator through the stochastic resonance algorithm. Since human vision is especially sensitive to moving images, it is still possible that the changing images obtained from the added noise and thresholding process will lower the detection threshold required for operators to identify contacts in sonar displays.

Now consider what happens when one starts with a 3 bit image with values between zero and seven. Again, each pixel of the image is assumed to contain either noise only or a combination of signal plus noise. If the pixel contains signal in addition to noise, the pdf will be displaced to the right as long as the signal level is at least one. From the argument above, we know that the process of thresholding may improve the SNR slightly, however, the question

remains as to whether the process of producing a movie of the image with varying added noise will improve the detectability of low level signals. Following the thought process of Marks et. al., we can ensure that the overall expected value of the pixel level after adding the noise will be the same as the original value by using a uniform noise distribution with a threshold set at half of the maximum pixel value. For a 3 bit image, the required noise distribution will be uniform between -3.5 and 3.5 with a threshold of 3.5. Letting x indicate the original pixel level, ξ, the added noise, Δ, the threshold, and z, the pixel level after adding noise and thresholding, and the expected value of z is given by:

$$E\left[z\right]=\Pr\left[x+\xi-\Delta>0\right]=\Pr\left[\xi>\Delta-x\right] \qquad \textbf{Eqn. 8}$$

.

For $\Delta=3.5$ and ξ uniform between -3.5 and 3.5, we find that:

$$E\left[z\right]=\Pr\left[x+\xi-3.5>0\right]=\Pr\left[\xi>3.5-x\right]=\int_{3.5-x}^{3.5}dx'=x \qquad \textbf{Eqn. 9}$$

This combination of noise distribution and threshold has the advantage that it will not result in an image which is degraded relative to the original by the addition of noise, since the average tends to converge back to the original image.

IV. EXPERIMENTAL TRIALS

A. PRELIMINARY EXPERIMENTS

In order to examine the performance of SR image processing, we first intended to apply the SR algorithm to an image of a real signal. We collected the signal from a passing airplane, as shown in Figure 1. The goal was to apply the SR algorithm to this image in order to improve the visibility of the low-level tonals. A tool that we created to aid in our analysis was a graphical user interface (GUI) in MATLAB. The GUI allows us to add and subtract noise, as well as to raise and lower the threshold, by adjusting two slide bars. The GUI enables us to visually observe improvement, but it does not help us to quantify the improvement. After creating the GUI, it became apparent that we needed to begin with a simulated signal so that an initial and final signal-to-noise ratio could be calculated. This would enable us to quantitatively determine the effect of SR image processing on the resulting image.

The first standard of measurement we used to calculate the quality of the images was the peak signal-to-noise ratio, introduced in Chapter II. This method involved comparing the final SR image with an initial noise-free image. This was an effective method for comparing simulated signals, but it produced results that were nearly impossible to apply to real-world situations, since we would never have a noise-free image available.

Our final approach was to create a simulated signal in noise, and to calculate its signal-to-noise ratio. We could then examine how the noise and threshold values affected the final image quality.

B. HISTOGRAMS

To get a better idea of what happens to individual pixels during the stochastic resonance process, it is helpful to look at histograms of an image. A histogram graphs the pixel values vs. the number of pixels that have that value.

The following demonstration will use histograms to illustrate how the stochastic process may be used to improve an image. The success of the method depends on, among other things, the level of the signal and the amount of background noise. In this case we will use a convenient signal amplitude and noise level in order to illustrate the steps of the process.

Consider an image that contains 500 pixels. We can start by looking at a histogram of this image if it contained only Gaussian distributed noise with a mean of $\mu = 2$ and a standard deviation of $\sigma = 1$.

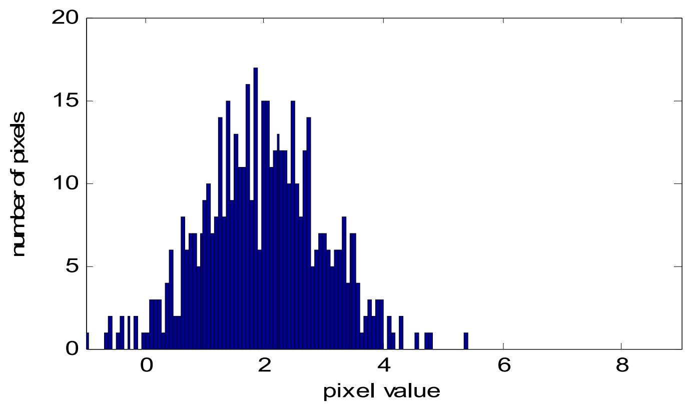


Figure 17. Histogram of normally distributed noise: $\mu=2$, $\sigma=1$

If a signal is also present with an amplitude of 3, the resulting histogram would have a mean of $\mu = 5$ and a standard deviation of $\sigma = 1$.

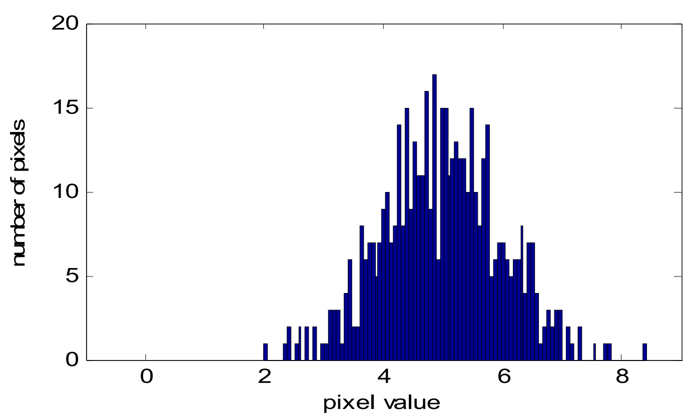


Figure 18. Histogram of signal of amplitude 3 in the presence of normally distributed noise

Next, we can look at the histogram of the image after the first stochastic resonance step, which is adding random noise to every pixel. In this case, we will add uniform random noise from -1 to 1. Note that this image with additional noise is only one of twenty that will be thresholded and averaged together.

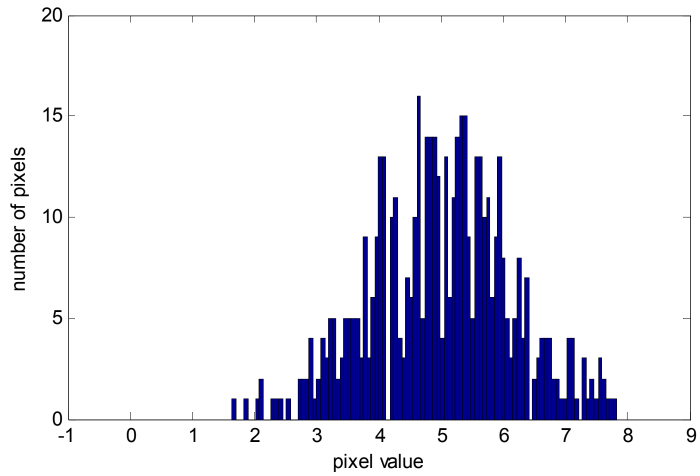


Figure 19. Histogram of image after adding uniformly distributed random noise to every pixel

Now we can complete the process by applying a threshold and averaging. The final histogram for the image that contains only noise will be compared to the histogram for the image that contains both noise and a signal. Figure 20 is the histogram of the final image after the SR process is done on the pixels that contain only noise. The threshold level was four. Fewer bins are used in this histogram in order to demonstrate the range of the pixels.

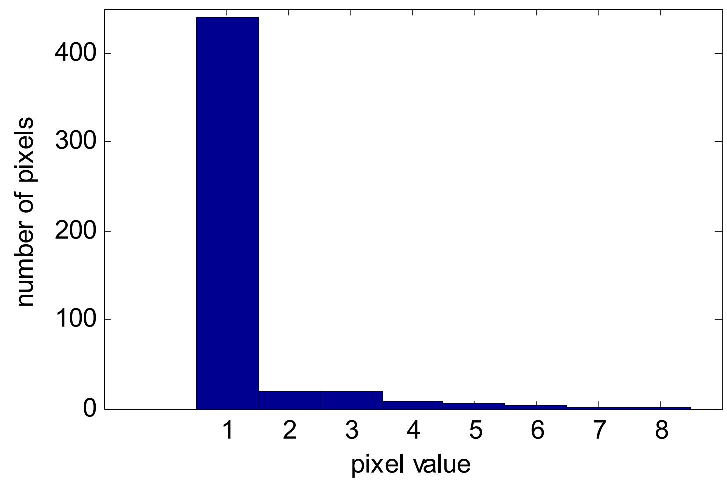


Figure 20. Histogram of final SR image after applying threshold at Δ=4; noisy pixels only using uniformly distributed random noise

The histogram in Figure 21 is for the final image after the SR process is done on the pixels that contain both noise and a signal.

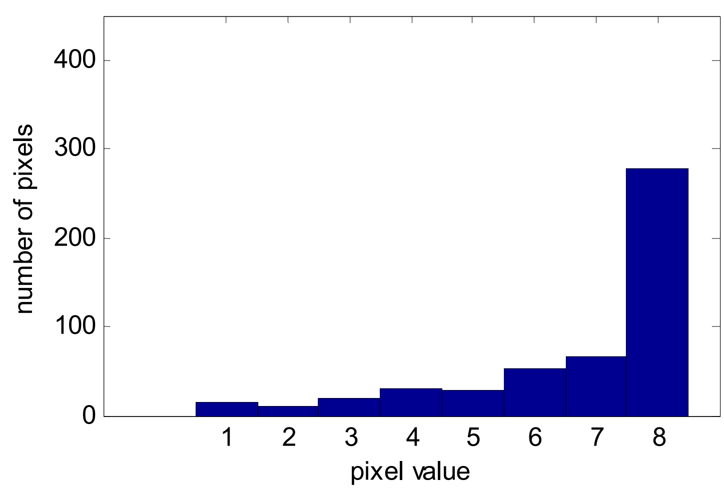


Figure 21. Histogram of final SR image after applying threshold at Δ=4; noise and signal using uniformly distributed random noise

From the histograms, one can see that when no signal is present, less than one percent of the pixels become black, whereas when a signal is present, 57% of the pixels become black. Also, in the signal-free image, 97% of the pixels

ended up with a value less than or equal to four, whereas in the image with the signal, only 16% of the pixels had values in that range. This example demonstrates how stochastic resonance can make signal pixels stand out compared to pixels where only noise is present. As stated earlier, the values of μ, α, and Δ were chosen for convenience to illustrate how the process may work in favorable conditions. In this case, the pixel level was two levels above the mean of background noise. In a real LOFARgram, the signal may not be as distinguishable as this. Chapter IV will discuss how the stochastic resonance method works in more realistic circumstances.

Next, we can do the same experiment using Gaussian noise, in order to determine which noise distribution is more effective. Again, we will begin with a simulated signal that is two levels above the background noise. The histograms for the background noise and for the background noise plus the signal are the same as Figure 17 and Figure 18. After adding randomly distributed Gaussian noise with σ =1, the resulting histogram is displayed in Figure 22.

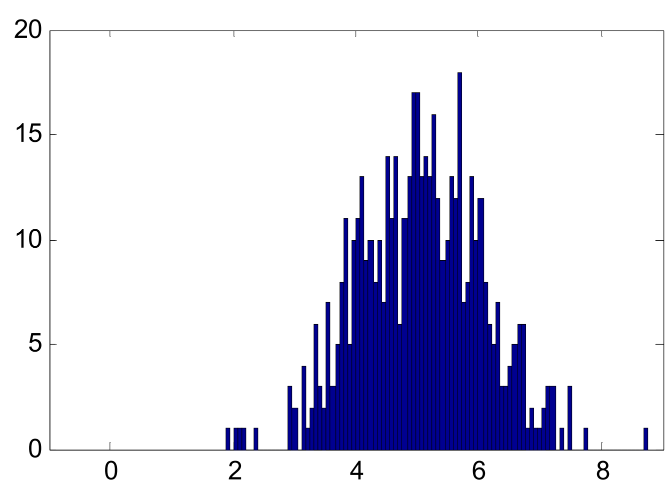


Figure 22. Histogram of image after adding Gaussian distributed random noise to every pixel

Figure 23 is the histogram of the final image after the SR process is done on the pixels that contain only noise.

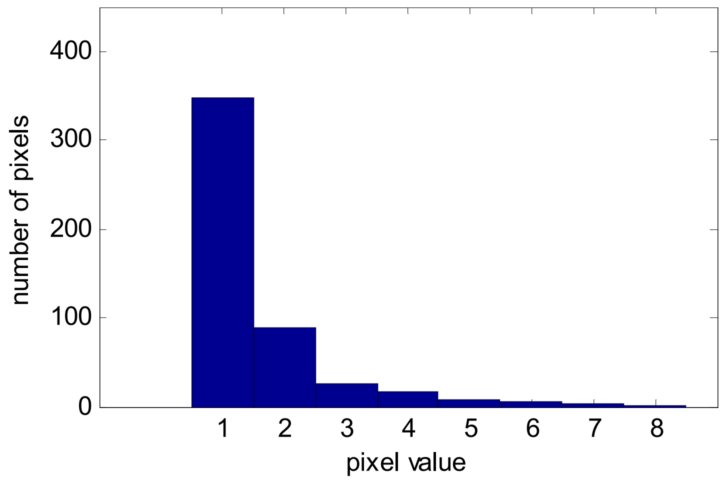


Figure 23. Histogram of final SR image after applying threshold at ∆=4; noisy pixels only using Gaussian distributed random noise

The histogram in Figure 24 is for the final image after the SR process is done on the pixels that contain both noise and a signal.

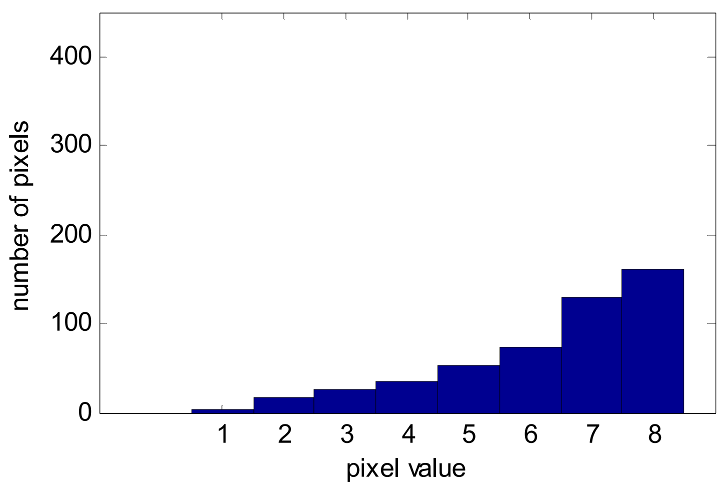


Figure 24. Histogram of final SR image after applying threshold at ∆=4; noise and signal using Gaussian distributed random noise

We can compare the results for uniformly distributed noise with those for Gaussian distributed noise in order to decide which distribution is more effective for our purposes. Table 1 compares the results for both uniform and Gaussian noise, for the pixels containing only noise, and those containing both noise and a signal.

Pixel Value	Uniform Noise		Gaussian Noise	
	Noise Only (%)	Signal and Noise (%)	Noise Only (%)	Signal and Noise (%)
1	86	4	67	1
2-4	11	12	30	16
5-7	3	27	3	48
8	< 1	57	< 1	35

Table 1. Resulting pixel values compared for uniform and Gaussian noise

From the previous histograms and Table 1, it is clear that adding uniformly distributed random noise is more favorable than adding Gaussian distributed noise. Adding uniformly distributed noise causes a greater percentage of signal-containing pixels to consistently cross the threshold than Gaussian noise. Likewise, uniform noise results in fewer signal-free pixels crossing the threshold on average. Therefore, in this thesis, uniformly distributed noise will be used in the stochastic process rather than Gaussian distributed noise.

C. MEASURING DETECTABILITY

We will start with a very basic simulated signal of constant amplitude with no background noise. Using a gray scale from 1(white) to 8 (black), the simulated signal will have an amplitude of 1 and will be in the presence of background noise with a mean of $\mu=2$ and a standard deviation of $\sigma = 1$. This simulated signal is barely visible.

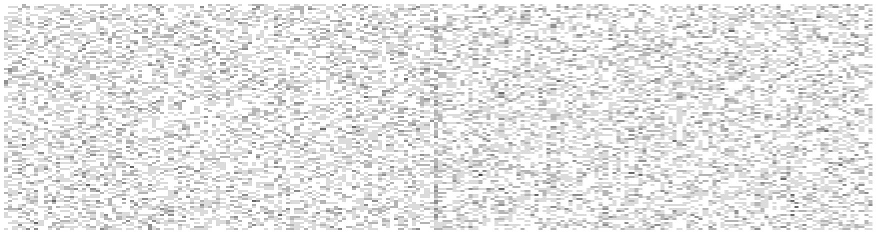


Figure 25. Simulated signal of constant amplitude 1 with background noise: μ=2, σ = 1

We must now find the optimal noise and threshold values to use in the stochastic process to improve the detectability of this signal. In order to accomplish this, we must establish a way to measure the visibility of the signal. The steps we will use to calculate the detectability of the signal are as follows:

1. Add together the pixel values in each column of the image. The simulated image above contains 200 columns, with the signal in the 100th column. Therefore, summing together the pixel values in each column will result in a vector of 200 values.

2. Normalize the sum vector by dividing each value by the maximum value in the vector.

3. Graph the sum vector vs. its indices. Figure 26 is the graph for the above noisy image.

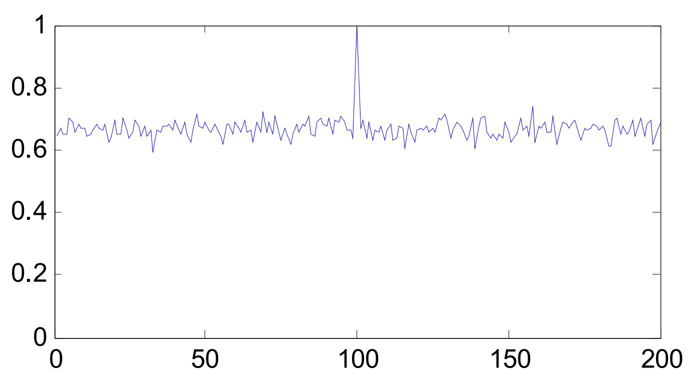


Figure 26. Normalized sum vector for original noisy image

The spike on the graph represents the column that contains the signal. To measure the quality of the image, we will calculate the signal to noise ratio using **Eqn. 10**

$$SNR = \frac{1}{E[x^2]} \qquad \textbf{Eqn. 10}$$

where

$$E[x^2] = \frac{\sum_{x=i}^{N} x^2}{N} \qquad \textbf{Eqn. 11}$$

is the expectation of the square of the background noise. The signal-to-noise ratio will be the measurement used to compare the detectability of signals in noisy images.

D. FINDING OPTIMAL NOISE AND THRESHOLD VALUES

Now that we have a way to measure the quality of an image, we can use this to determine optimal noise and threshold values. Recall that the symbols α and Δ are used to represent the amount of noise added and the threshold value, respectively. Since we are adding uniformly distributed noise, -α and α are the lower and upper bounds of the distribution.

In order to carry out this investigation, we will plug in reasonable values of α and Δ, and calculate the resulting SNR. The values that produce the highest signal ratio should be the optimal noise and threshold values. For the noisy image above, the average SNR is 2.21.

In order to find the optimal noise and threshold values, SNR was calculated for values of Δ from 0 to 6 and values of α from 0 to 2 in increments of 0.25. The results are shown in Figure 27.

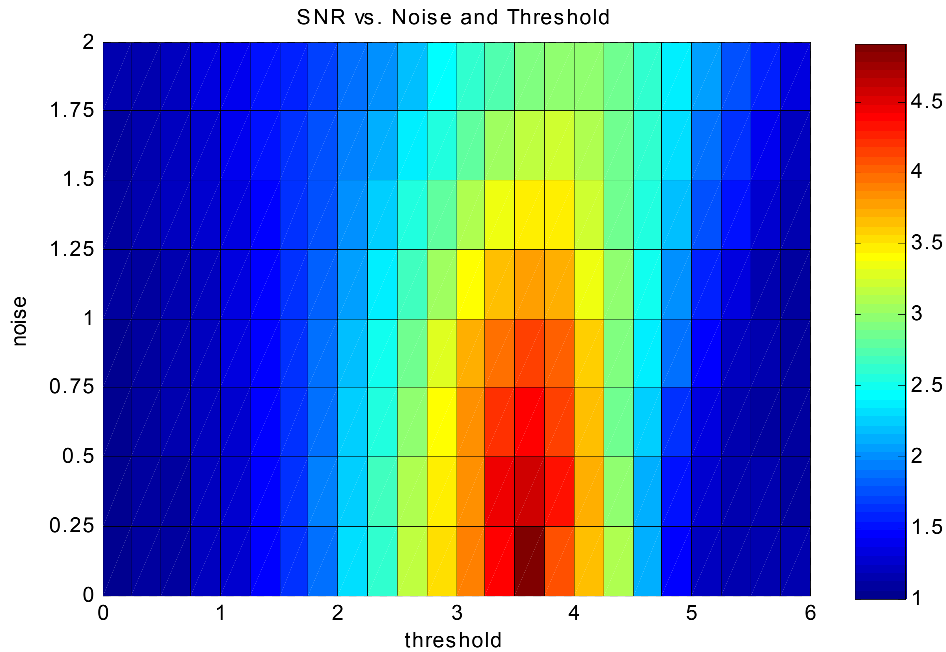


Figure 27. SNR vs. Noise and Threshold

From this graph, it appears that the optimal threshold value is Δ=3.5, and the optimal value for α is between zero and 0.25. A graph with finer resolution shows that the peak SNR value is found when α=0. This confirms our prediction, that applying a threshold without adding additional noise produces the best result. As an example, applying the values of α=0.2 and Δ=3.5 to our original noisy image results in the improvement shown by the sum vector graphs in Figure 28 and the images in Figure 29.

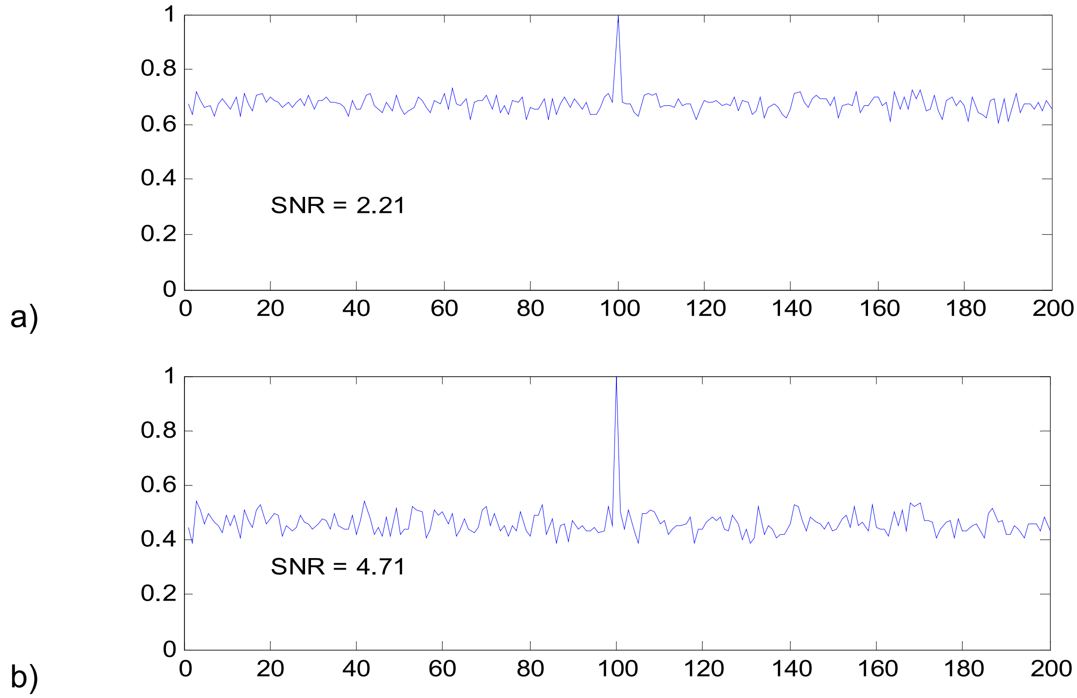


Figure 28. Normalized sum vectors for (a) initial noisy image and (b) final image using α=0.2 and Δ=3.5

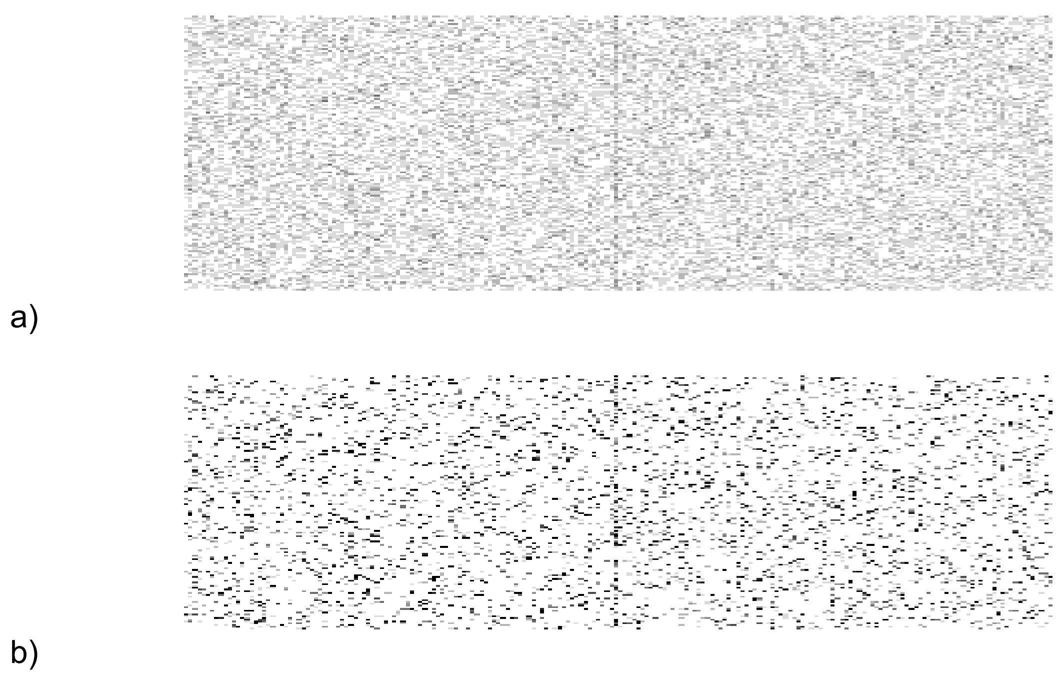


Figure 29. (a) initial noisy image and (b) improved image using α=0.2 and Δ=3.5

The average difference between the initial and final SNR after 50 trials was 1.92, with the minimum improvement being 1.20 and the maximum being 2.72. As expected, the improvement was slightly better when adding no noise at all.

E. A MORE ACCURATE LOFAR SIMULATION

In order to apply the SR process to real LOFARgrams, we must first rectify some differences between our experiments and real-world data. The main difference is that so far, the images we have been using have contained fractional pixel values. In a real 3-bit LOFARgram, the pixel values will be integers. This next experiment will demonstrate the SR process using a simulated signal image produced by a MATLAB program from JHU-APL. The program requantizes an image to produce realistic LOFARgram simulations. Figure 30 contains signals of varying amplitude in the presence of background noise. The image is 500 pixels by 1024 pixels. In order to perform the SR process on this sample LOFARgram, we will first isolate a weak signal from the image. The chosen signal is in the 221st pixel column, and is not visible in Figure 30.

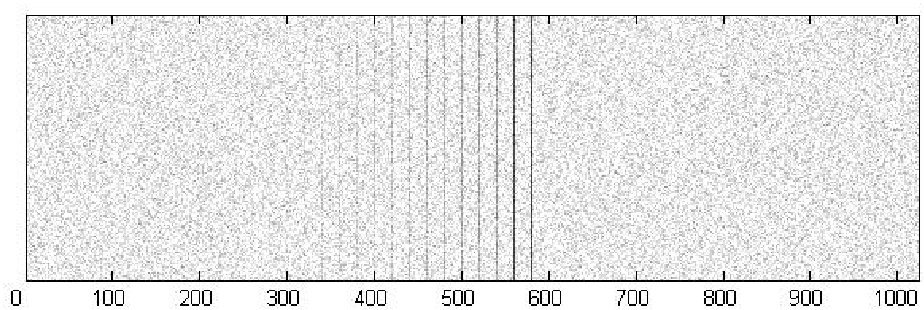


Figure 30. Initial image generated by LOFAR_Simple_Driver program

Figure 31 displays the image after the weak signal has been isolated, and Figure 32 graphs the sum vector for this image. The arrow in Figure 31 indicates the location of the weak signal.

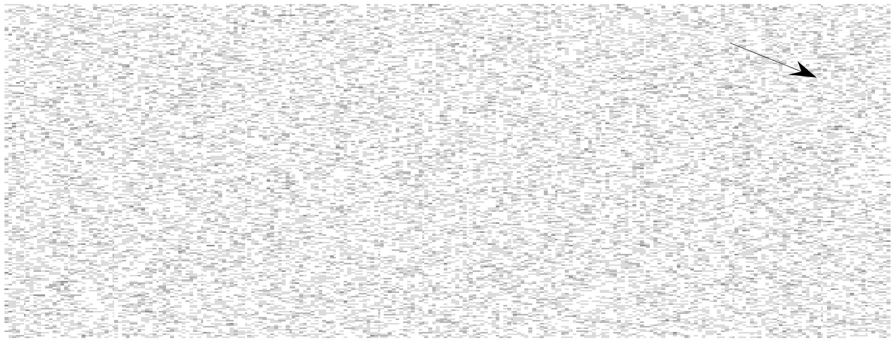


Figure 31. Isolated weak signal

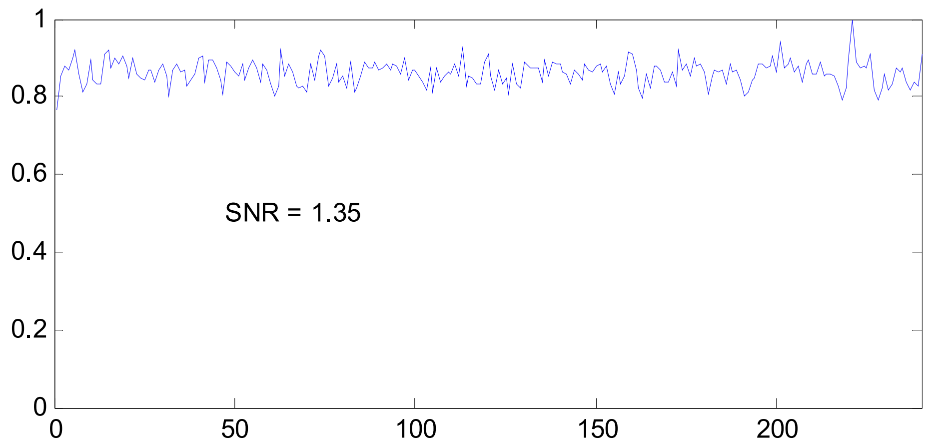


Figure 32. Graph of the sum vector for the isolated weak signal

Next, we will apply the SR process to this image in an attempt to improve the visibility of the weak signal in the 221st column. Again, SNR is calculated using values of α from 0 to 2, and values of Δ from 0 to 6. This produces the topographic plot in Figure 33.

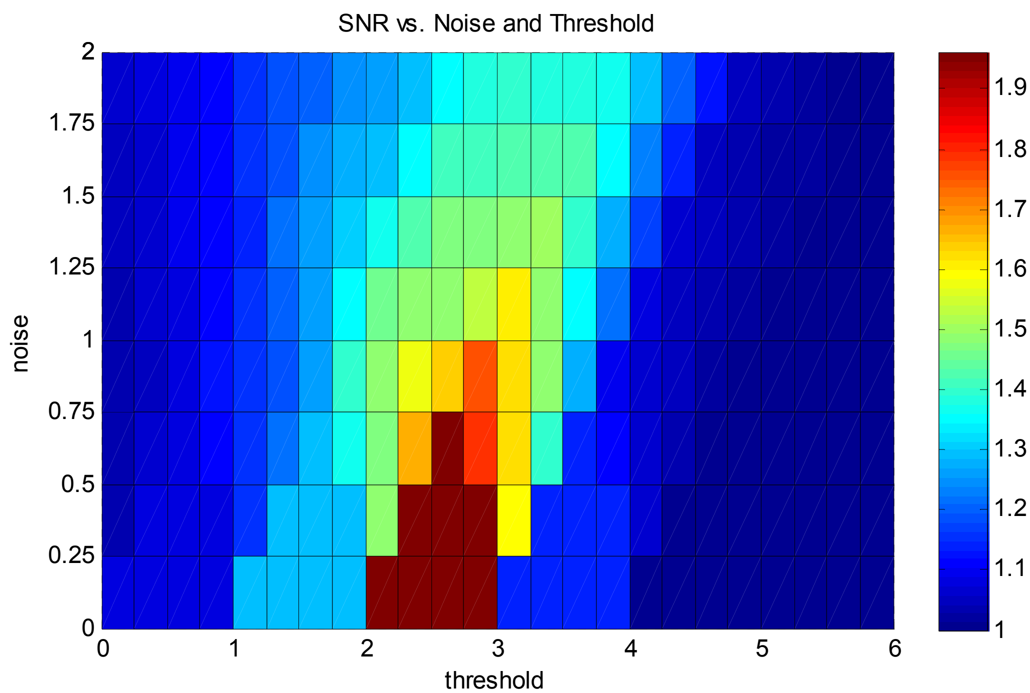


Figure 33. SNR vs. Noise and Threshold

From this graph, one can see the optimal values of noise and threshold are in the range $0<\alpha<0.75$ and $2<\Delta<3$. Since the original pixel values are all integers, adding noise with $\alpha<0.5$ produces the same result as applying a threshold without adding any noise at all. In the case where no noise is added, each pixel will either always cross the threshold, or never cross the threshold. Therefore, since only one thresholded image is created, no averaging can take place. Also, no movies can be made, since all of the images created will be identical.

Applying the values $\alpha=0$ and $\Delta=2.5$ to the image of the isolated weak signal results in the following improvements. Figure 34 is the improved image, and Figure 35 is the graphed sum vector for the same image.

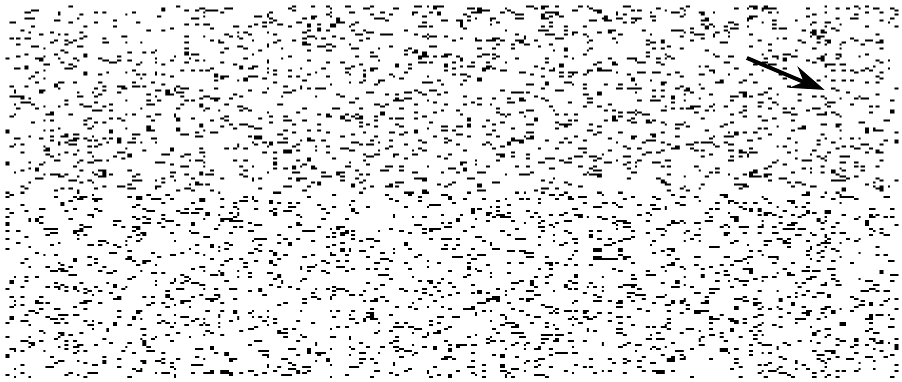


Figure 34. Isolated signal after applying optimal values of α=0 and Δ=2.5

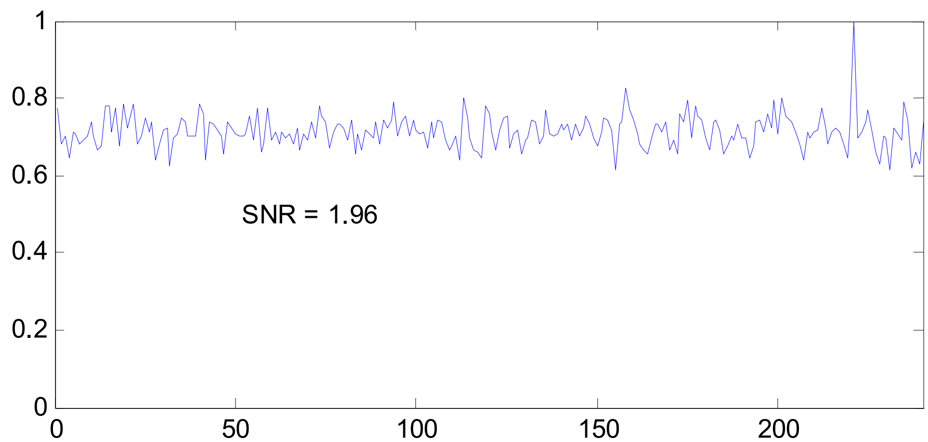
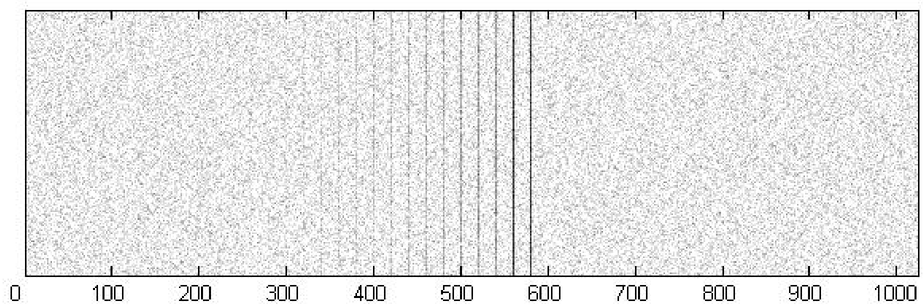


Figure 35. Graph of sum vector for improved image

Although the SNR has improved due to the SR process, the signal is still not visible in the image. Note that If the values α=3.5 and Δ=3.5 were used, the final SNR would equal the original SNR.

This experiment was repeated for the slightly stronger signal in column 241 of the initial image, and an identical range of optimal values was found. This tells us that to illuminate weaker signals, the best possible image is produced in the range 0<α<0.75 and 2<Δ<3. Values within this range were applied to the original 500x1024 image. Figure 36 shows the original image directly above the best possible improved image.

a)



b)

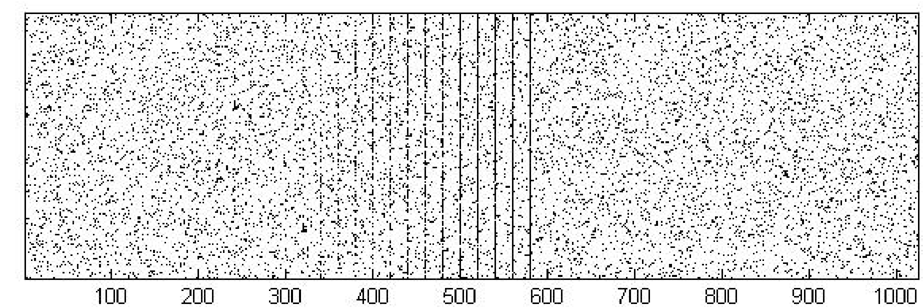


Figure 36. a) original image and b) improved image using α=0 and Δ=2.5

It appears that thresholding can be used to improve the contrast between visible lines and background noise, but that lines that are hidden in background noise in an original image are irrecoverable using this process. We have also verified in this chapter that thresholding an image without previously adding noise always produces a better image than thresholding an image after adding noise.

V. RESULTS AND RECOMMENDATIONS

At this point, we can conclude that the SR process does not produce a measurable improvement in the visibility of weak signals when the SR images are averaged together into a final image. It can also be stated that since the optimal value for α was found to be zero in every case, that any SNR improvement found during our experiments cannot be considered stochastic resonance. It is merely a result of applying a threshold that maximizes the number of signal-containing pixels that go to black and the number of noise-only pixels that go to white. Therefore, the only possible benefit from the SR process would have to be found by making movies of the images, rather than averaging them.

Making movies of SR images presents an additional challenge, in that it is difficult to quantify any improvement which might be realized due to stochastic resonance in the optical nerves. For the purpose of this thesis, several movies were made of SR images and observed qualitatively. It appears that there may be some additional benefit in making movies over viewing static averaged images. In order to measure this improvement, it would be necessary to use human test subjects, as was done in Simonotto's experiment.

For a follow-up experiment, the images used should be as similar to real LOFARgrams as possible. This means that the color scale should be identical, and the background noise intensity and distribution should be realistic. A good image to use might be similar to the image in Figure 30. A MATLAB code for making SR movies is provided in Appendix A. The best human test subjects to use would be experienced sonar operators. The subjects could be shown a series of movies, with each movie using a different combination of α and Δ in the SR process. The subjects would be asked to point out the weakest signal that they can detect in each movie. As discussed in Chapter III, we suspect that the best results may be found by displaying series of SR images in which α and Δ are both equal to one half of the maximum pixel value of the grayscale. After

many trials are completed by several test subjects, conclusions could be drawn regarding whether or not stochastic resonance movies improve the detectability of weak signals, and if so, what values of α and Δ are optimal.

APPENDIX .   MATLAB CODE FOR SR MOVIES

```
clear
n=3.5;
t=3.5;

c = linspace(1,0,8);
C = [c;c;c]';
colormap(C);

      %any image matrix lofX containing pixel values from 0 to 7 can be
      used for plane3.mat

load plane3.mat
Mnois=lofX;
[j,k]=size(Mnois);

            for i=1:10;
                noise=(n+(-2*n).*rand(j,k));
                B=Mnois+noise;
                Mth=(B>t)*7;
                image(Mth);
                F(i)=getframe;
            end

figure(1)
colormap(C);
movie(F,5,60)
axis off
```

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